SOLUTION SET

Chapter 8

LASER OSCILLATION ABOVE THRESHOLD

"LASER FUNDAMENTALS"

Second Edition

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CH 8

1. An argon ion laser (as shown in Figure 10-5(b)) operating at 488.0 nm with a gain-region length of 0.3 m is determined to have a population density of $1 \times 10^{16}/\text{m}^3$ in the upper laser level and $4 \times 10^{15}/\text{m}^3$ in the lower laser level. If a very low-power probe beam of intensity I_0 is transmitted into the gain medium then what would be the measured ratio of I/I_0, where I is the output beam intensity after traversing the medium?

$\lambda = 488.0\,\text{nm}$ $\quad L = 0.3\,\text{m}$

$N_u = 1 \times 10^{16}/\text{m}^3$ $\quad N_e = 4 \times 10^{15}/\text{m}^3$

$$I = I_0 e^{\sigma_0 [N_u - (\frac{g_u}{g_e}) N_e] L}$$

$\sigma = 2.5 \times 10^{-16}\,\text{m}^2$ From Fig. 7-10 pg 246

$g_u = 2J_u + 1 = 2(5/2) + 1 = 6$

$g_e = 2J_e + 1 = 2(3/2) + 1 = 4$

$$\frac{I}{I_0} = e^{(2.5 \times 10^{-16})(1 \times 10^{16} - \frac{6}{4} 4 \times 10^{15})(0.3)}$$

$$= e^{0.3} = \underline{1.35}$$

2. What is the small-signal gain coefficient of the laser described in Problem 1?

$$g_0 = \sigma^D \Delta N$$

$$= (2.5 \times 10^{-16}) \left[1 \times 10^{16} - \left(\frac{6}{4}\right) 4 \times 10^{15} \right]$$

$$= \underline{1.00/m}$$

3. What would be the threshold gain of the laser described in Problem 1 if the mirrors have reflectivities of 99.9% and 95% and if the scattering losses are 0.5% per pass? Assume no distributed absorption within the gain medium.

$\lambda = 488.0\ nm \qquad \alpha = 0 \qquad L = 0.3\ m$

$R_1 = 0.999 \qquad R_2 = 0.95 \qquad a_1 = a_2 = 0.005$

$$g_{Th} = \frac{1}{2L} \ln\left[\frac{1}{R_1 R_2 (1-a_1)(1-a_2)}\right]$$

$$= \frac{1}{2(0.3)} \ln\left[\frac{1}{(0.999)(0.95)(1-.005)(1-.005)}\right]$$

$$= \frac{1}{0.6} \ln 1.0643 = \frac{6.23 \times 10^{-2}}{0.6}$$

$$= 1.039 \times 10^{-1}/m = \underline{0.1039/m}$$

4. What would be the power output from the laser described in Problems 1–3?

$$I_T = \frac{I_{sat}}{2}\left[\left(\frac{g_0 L}{a+T}\right) - 1\right] T$$

$$g_0 = 1/m \qquad T = \frac{\overbrace{(1-0.999)(1-0.95)}^{\text{average}}}{2} = 2.55\times10^{-2}$$

Assume 1mm dia beam

$$\text{Area} = \pi(0.0005)^2 = 7.85\times10^{-7}\,m^2$$

$$a = 0.005$$

$$I_{sat} = \frac{h\nu}{\sigma^H \tau_u}$$

$$h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,Js\;\; 3\times10^{8}\,m/s}{488\times10^{-9}\,m} = 4.08\times10^{-19}\,J$$

$$\sigma^H = \frac{\lambda^2 A_{ul}}{4\pi^2 n\,\Delta\nu_{ul}^H}$$

From Table 14-2

$$A_{ul} = 7.8\times10^{7}$$

$$n = 1$$

$$\tau_u = 1.28\times10^{-8}$$

$$\Delta\nu_{ul}^H = \frac{\left[\sum_i A_{ui} + \sum_j A_{lj}\right]}{2\pi}$$

From Fig 10-5b

$$= \frac{[7.8\times10^{7} + 23\times10^{8} + 4.5\times10^{8}]}{2\pi} = 4.50\times10^{8}/s$$

$$\therefore\ \sigma^H = \frac{(488\times10^{-9})^2\, 7.8\times10^{7}}{4\pi^2\, 4.5\times10^{8}} = 1.05\times10^{-15}\,m^2$$

$$\therefore\ I_{sat} = \frac{4.08\times10^{-19}\,J}{1.05\times10^{-15}\,m^2\; 1.28\times10^{-8}\,s} = 3.04\times10^{4}\,W/m^2$$

$$I_T = \frac{3.04\times10^{4}}{2}\left[\frac{1\,(0.3)}{0.005+0.0255} - 1\right] 2.55\times10^{-2}$$

$$= 3.42\times10^{3}\,W/m^2$$

Double the output for one end

$$\text{Power} = (2)\,3.42\times10^{3}\,W/m^2 \cdot 7.85\times10^{-7}\,m^2 = 5.37\times10^{-3}\,W$$

$$= \underline{5.37\,mW}$$

5. A low-signal probe beam of the same wavelength as the laser is passed through the gain medium of an 8-mm–diameter Nd:YAG laser rod that is flashlamp pumped. The probe beam with input intensity of I_0 is measured to be a factor of 10 higher when it emerges from the gain medium ($I/I_0 = 10$). The same laser rod (under the same pumping conditions) is used as an amplifier to increase the power of another Nd:YAG beam. If the input intensity from that other laser is a factor of 10 above I_{sat} when it enters the amplifier, what would be the power emerging from the output end of the laser rod? Assume that the beam is the same diameter as the rod.

I_0 = low power probe beam

8 mm dia Nd:YAG laser rod

$$\frac{I}{I_0} = 10 = e^{gL} \Rightarrow gL = \ln 10 = 2.30$$

High power beam passed through same rod

Input of that beam is 10 I_{sat} or

$$I_{in} = 10\, I_{sat}$$

from (8.15) we have

$$I = I_{in} + gL\, I_{sat} \quad \text{or}$$

$$I = 10\, I_{sat} + 2.30\, I_{sat} = 12.30\, I_{sat}$$

From Table 15-5

$\sigma H = 2.8 \times 10^{-23}\ m^2$

$\tau = 230\ \mu s$

$\lambda = 1.06\ \mu m$

$$I_{sat} = \frac{h\nu}{\sigma H \tau} = \frac{hc}{\sigma H \tau \lambda}$$

$$= \frac{6.63\times10^{-34}\ Js\ \ 3\times10^{8}\ m/s}{2.8\times10^{-23}\ m^2\ 2.3\times10^{-4}\ s\ \ 1.06\times10^{-6}\ m} = 2.91\times10^{7}\ W/m^2$$

$$\therefore\ I = 12.30\, I_{sat} = 3.58\times10^{8}\ W/m^2$$

for rod of diameter 8 mm, area $= \pi(0.004)^2 = 5.03\times10^{-5}\ m^2$

$$\therefore\ \text{Power} = I \times \text{area} = 3.58\times10^{8}\ \frac{W}{m^2} \times 5.03\times10^{-5}\ m^2$$

$$= \underline{1.80\times10^{4}\ W}$$

6. A pulsed Nd:YAG laser rod of length 0.1 m and diameter 8 mm is flashlamp pumped and is measured to have a single-pass gain (I/I_0) of 5 at the laser wavelength of 1.06 μm. The pumping duration and thus the output pulse is several hundred microseconds, so we may think of it as a quasi-continuous beam. The rod also has measured scattering losses per pass of 0.5%. Mirrors are installed at each end of the rod. What would be the optimum transmission for the output mirror if a high-reflecting mirror is used at one end of the cavity and the output mirror at the other end? What would be the power output with this mirror combination? Assume the laser beam is the same diameter as the laser rod.

$L = 0.1\,m \quad d = 8\,mm \quad \lambda = 1.06\,\mu m$

scattering losses $a = 0.005/pass$

$$\frac{I}{I_0} = 5 \quad \text{(single pass gain)}$$

$$T_{opt} = (g_0 L a)^{1/2} - a$$

$$\frac{I}{I_0} = 5 = e^{g_0 L} \Rightarrow g_0 L = \ln 5 = 1.609$$

$$\therefore \; T_{opt} = \left[(1.609)(0.005)\right]^{1/2} - 0.005 = 8.47 \times 10^{-2}$$

<u>Double T_{opt} for one output mirror</u>

$$P_{opt} = I_T^{opt} \times Area = \left(\frac{T_{opt}^2}{2a}\right) I_{sat} \times Area \quad \text{(from (8.27))}$$

$$= \frac{(8.47 \times 10^{-2})^2}{2\,(0.005)} I_{sat} \times Area = 0.717\, I_{sat} \times Area$$

From problem 8-5 $\quad I_{sat} = 2.91 \times 10^7\, W/m^2$

$$\therefore P_{opt} = (0.717)(2.91 \times 10^7\, W/m^2) \times \pi (0.004)\, m^2$$

$$= \underline{1.05 \times 10^3\, W}$$

Double This for power from one end = <u>2,100 W</u>

7. What would be the output intensity of a helium–neon laser operating at 632.8 nm if it has a small-signal gain of 0.15/m? The laser has two mirrors of reflectivity 99.99% and 98%, a gain-medium length of 0.2 m, and essentially no scattering losses.

$g_0 = 0.15/m \qquad R_1 = 0.9999 \qquad R_2 = 0.98$

$L = 0.2\,m \qquad a = 0$

$$I_T = \frac{I_{sat}}{2}\left[\left(\frac{g_0 L}{a+T}\right) - 1\right]T \quad \text{for } R_1 = R_2$$

average of $R_1 + R_2$ is $\frac{R_1+R_2}{2} = 0.9895$

$g_0 L = (0.15/m)(0.2\,m) = 0.03$

and $T = 1 - 0.9895 = 0.0105$

$$I_{sat} = \frac{h\nu}{\sigma^H \tau_u} = \frac{hc}{\sigma^H \tau_u \lambda}$$

$$\sigma^H = \frac{\lambda^2 A_{ul}}{4\pi^2 \Delta\nu_{ul}^H} = \frac{\lambda^2 A_{ul}}{4\pi^2 \Delta\nu^H}$$

From Table 4-1 $A_{ul} = 3.4\times10^6 \quad \Delta\nu^H = 1.4\times10^7$

$$\therefore \sigma^H = \frac{(632.8\times10^{-9})^2\, 3.4\times10^6}{4\pi^2\; 1.4\times10^7} = 2.46\times10^{-15}\,m^2$$

$$I_{sat} = \frac{(6.63\times10^{-34}\,J\,s)\,3\times10^8\,m/s}{(2.46\times10^{-15}\,m^2)\,3\times10^{-8}\,s\;\; 632.8\times10^{-9}\,m} = 4.26\times10^3\,\frac{W}{m^2}$$

$$\therefore I_T = \frac{4.26\times10^3\,\frac{W}{m^2}}{2}\left[\frac{0.03}{0+0.0105} - 1\right](0.0105)$$

$$= \underline{41.54\,W/m^2}$$

Double This for one output mirror $\Rightarrow$ $\underline{83.1\,W/m^2}$

8. Obtain (8.100) and (8.101) from the general expression of (8.99) by using approximations for low initial beam energy ($E_0/E_{\text{sat}} \ll 1$) and high initial beam energy ($E_0/E_{\text{sat}} \gg 1$).

(a) $$F = F_{sat} \ln\left\{1 + e^{g_0 L}\left[e^{F_0/F_{sat}} - 1\right]\right\}$$

for $\frac{F_0}{F_{sat}} << 1$ $\quad e^{F_0/F_{sat}} \cong 1 + \frac{F_0}{F_{sat}}$

$$\therefore \frac{F}{F_{sat}} = \ln\left\{1 + e^{g_0 L}\left[1 + \frac{F_0}{F_{sat}} - 1\right]\right\}$$

$$= \ln\left[1 + \frac{F_0}{F_{sat}} e^{g_0 L}\right]$$

$$e^{F/F_{sat}} = 1 + \frac{F_0}{F_{sat}} e^{g_0 L}$$

$$\cong 1 + \frac{F}{F_{sat}} = 1 + \frac{F_0}{F_{sat}} e^{g_0 L}$$

$$\therefore \underline{F = F_0 e^{g_0 L}}$$

(b) for $\frac{F_0}{F_{sat}} >> 1$ $\quad e^{F_0/F_{sat}} - 1 \cong e^{F_0/F_{sat}}$

$$\therefore F = F_{sat} \ln\left\{1 + e^{g_0 L} e^{F_0/F_{sat}}\right\}$$

$$= F_{sat} \ln\left\{1 + e^{(g_0 L + F_0/F_{sat})}\right\}$$

$$\cong F_{sat} \ln\left\{e^{(g_0 L + F_0/F_{sat})}\right\}$$

$$= F_{sat}(g_0 L + F_0/F_{sat})$$

$$\therefore \underline{F = F_0 + g_0 L F_{sat}}$$

9. Determine the saturation energy of the following lasers:

(a) Nd:YAG at 1.06 μm;
(b) Ti:Al_2O_3 at 800 nm;
(c) ruby at 694.3 nm;
(d) Nd:YVO_4 at 1.06 μm.

$$F_{sat} = \frac{h\nu}{\sigma^H} \qquad h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,3\times10^{8}}{\lambda} = \frac{1.99\times10^{-25}}{\lambda}$$

Laser	λ (μm)	$h\nu$ (J)	σ^H (m^2)	F_{sat} (J/m^2)
Nd:YAG	1.06	1.88×10^{-19}	2.8×10^{-23}	6.71×10^{3}
Ti:Al_2O_3	0.800	2.49×10^{-19}	3.4×10^{-23}	7.32×10^{3}
Ruby	0.6943	2.87×10^{-19}	2.5×10^{-24}	1.15×10^{5}
Nd:YVO_4	1.06	1.88×10^{-19}	1.14×10^{-22} (π)	1.65×10^{3}

↑ From Table 15-7

10. A Nd:YVO_4 laser operating in a pulsed mode at 1.06 μm is placed in a cavity with mirrors of 95% reflectivity at each end of the laser. The scattering losses are determined to be 0.4% per pass. If the laser produces output pulses of 200-μs duration in a beam of diameter 6 mm and if the small-signal gain is measured to be 20% per pass, what would be the energy of the pulses emerging from the laser?

Nd: YVO_4 laser $\quad \lambda = 1.06\ \mu m$

$R_1 = R_2 = 0.95 \quad a_1 = a_2 = 0.004$

$T = 0.05$ at each end

$$\frac{I}{I_0} = 1 + 20\% = 1.2 = e^{g_0 L} \Rightarrow g_0 L = 0.182$$

$$I_T = \frac{I_{sat}}{2}\left[\left(\frac{g_0 L}{a+T}\right) - 1\right] T$$

$$I_{sat} = \frac{h\nu_{ul}}{\sigma^H \tau_u} = \frac{hc}{\sigma^H \tau_u \lambda_{ul}} = \frac{6.63\times10^{-34}\ 3\times10^{8}}{1.14\times10^{-22}\ 90\times10^{-6}\ 1.06\times10^{-6}}$$

$$= 1.83\times10^{7}\ W/m^2$$

$$\therefore\ I_T = \frac{1.83\times10^{7}}{2}\left[\left(\frac{0.182}{0.004+0.05}\right) - 1\right] 0.05$$

$$= 1.08\times10^{6}\ W/m^2$$

$$\text{Energy} = I_T \times \text{Area} \times \Delta t_{pulse}$$

$$= 1.08\times10^{6} \times \pi\,(0.003)^2\ 200\times10^{-6}\,s$$

$$= \underline{6.11\times10^{-3}\,J = 6.11\ mJ}$$

from each end of laser

11. A Nd:YLF amplifier operating at a wavelength of 1.047 μm was measured to provide an increase of a factor of 5 when a low-intensity probe beam was passed through it. A laser beam operating at the same wavelength with a pulse duration of 10 ns and an energy of 100 mJ is then injected into the amplifier. What would be the output energy of the beam when it emerges from the amplifier?

Nd: YLF $\quad \lambda = 1.047\ \mu m$

$$I = I_0 + 5 I_0 = 6 I_0$$

$$F_{sat} = \frac{h\nu}{\sigma H} = \frac{hc}{\sigma H \lambda} = \frac{6.63\times10^{-34}\ 3\times10^{8}}{1.8\times10^{-23}\ 1.047\times10^{-6}}$$

$$= 1.06\times10^{4}\ \frac{J}{m^2}$$

For a typical beam diameter of 3 mm

The saturation energy would be

$$1.06\times10^{4}\ \frac{J}{m^2}\ \pi\ (0.0015)^2 = \underline{0.075\ J}$$

Thus for a 100 mJ beam that is above the saturation energy, the beam would grow to $6 I_0$ or <u>600 mJ</u> according to (8.101)