SOLUTION SET

Chapter 9

REQUIREMENTS FOR OBTAINING POPULATION INVERSIONS

"LASER FUNDAMENTALS"

Second Edition

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1. Using the equations (9.8), (9.9), and (9.10) that were developed to describe changes in the populations of a three-level system, obtain expressions for the populations in the upper and lower laser levels, the inversion ratio, and the minimum pumping requirements given in (9.11), (9.12), (9.13), and (9.14).

$$\frac{dN_l}{dt} = -\Gamma_{li} N_l + \gamma_{il} N_i + \gamma_{ul} N_u = 0 \qquad (9.8)$$

$$\frac{dN_u}{dt} = -\gamma_{ul} N_u + \gamma_{iu} N_i = 0 \qquad (9.9)$$

$$\frac{dN_i}{dt} = -\gamma_{iu} N_i - \gamma_{il} N_i + \Gamma_{li} N_l = 0 \qquad (9.10)$$

from (9.9) $\gamma_{ul} N_u = \gamma_{iu} N_i$ (1)

from (9.10) $\Gamma_{li} N_l = (\gamma_{iu} + \gamma_{il}) N_i$ (2)

$$\frac{(1)}{(2)} = \frac{\gamma_{ul} N_u}{\Gamma_{li} N_l} = \frac{\gamma_{iu}}{\gamma_{iu} + \gamma_{il}} \Rightarrow \frac{N_u}{N_l} = \frac{\gamma_{iu} \Gamma_{li}}{\gamma_{ul}(\gamma_{iu} + \gamma_{il})} \qquad (9.13)$$

$N_i + N_u + N_l = N \Rightarrow N_i = N - N_u - N_l$ ← substitute in (1)

$$(\gamma_{ul} + \gamma_{iu}) N_u + \gamma_{iu} N_l = \gamma_{iu} N$$

from (9.13) $\gamma_{iu} N_l = \frac{\gamma_{ul}}{\Gamma_{li}} (\gamma_{iu} + \gamma_{il}) N_u$

$$(\gamma_{ul} + \gamma_{iu}) N_u + \frac{\gamma_{ul}}{\Gamma_{li}} (\gamma_{iu} + \gamma_{il}) N_u = \gamma_{iu} N$$

$$\therefore \quad \underline{N_u} = \frac{\Gamma_{li} \gamma_{iu}}{\Gamma_{li}(\gamma_{ul} + \gamma_{iu}) + \gamma_{ul}(\gamma_{iu} + \gamma_{il})} N \qquad (9.12)$$

and $$N_l = \frac{\gamma_{ul}(\gamma_{iu} + \gamma_{il})}{\gamma_{iu} \Gamma_{li}} \frac{\Gamma_{li} \gamma_{iu}}{\Gamma_{li}(\gamma_{ul} + \gamma_{iu}) + \gamma_{ul}(\gamma_{iu} + \gamma_{il})} N$$

$$\therefore \quad \underline{N_l} = \frac{\gamma_{ul}(\gamma_{iu} + \gamma_{il})}{\Gamma_{li}(\gamma_{ul} + \gamma_{iu}) + \gamma_{ul}(\gamma_{iu} + \gamma_{il})} N \qquad (9.11)$$

2. Referring to the example in Section 9.3 on the threshold pumping requirements for a ruby laser, estimate the number of pumping photons and the power required to pump the laser rod to the threshold value for a 6-mm–diameter, 0.1-m–long ruby laser rod doped to the concentration given in the example during a pumping cycle that lasts for the duration of the upper laser level lifetime of 3 ms. Assume that the laser is pumped by photons of wavelength 550 nm. Remember that $N_0\Gamma_{li}$ represents the number of ground-state Cr^{3+} ions pumped per cubic meter every second into the upper laser level.

Choose $N_0 = 1.6 \times 10^{25}/m^3 = N_\ell$

Threshold pumping condition:

$$\Gamma_{\ell i} > A_{u\ell} = \frac{1}{\tau_u} = \frac{1}{3 \times 10^{-3} s} = 3.33 \times 10^2 /s$$

Pumping flux

$$N_0 \Gamma_{\ell i} = N_\ell \Gamma_{\ell i} = 5.33 \times 10^{27} / m^3 s$$

at Threshold

from example on pgs 297-98

$$\text{photons/sec} = (\text{pumping flux}) \cdot (\text{Volume})$$

$$= (5.33 \times 10^{27} / m^3 s)\, \pi\, (3 \times 10^{-3} m)^2\, (0.1 m)$$

$$= 1.51 \times 10^{22} \text{ photons/s}$$

$$\#\text{ photons} = (1.51 \times 10^{22} \text{ photons/s})(3 \times 10^{-3} s)$$

$$= \underline{4.52 \times 10^{19} \text{ photons}}$$

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = (\#\text{ photons/s})\, h\nu$$

$$= (1.51 \times 10^{22} \text{ photons/s})(6.62 \times 10^{-34} J s) \frac{3 \times 10^8 m/s}{550 \times 10^{-9} m}$$

$$= 5.45 \times 10^3 J/s = \underline{5.45 kW}$$

3. Beginning with the rate equations given in (9.16) and (9.17), obtain the threshold inversion requirements for a three-level atomic laser as given in (9.21).

$$\frac{dN_u}{dt} = N_0 \Gamma_{0u} - N_u(\gamma_{ul} + \gamma_{u0}) = 0 \qquad (9.16)$$

$$\frac{dN_l}{dt} = N_0 \Gamma_{0l} + N_u \gamma_{ul} - N_l \gamma_{l0} = 0 \qquad (9.17)$$

$$N_u = \frac{N_0 \Gamma_{0u}}{\gamma_{ul} + \gamma_{u0}} \qquad (9.18)$$

$$N_l = \frac{N_0 \Gamma_{0l} + N_u \gamma_{ul}}{\gamma_{l0}} = \frac{N_0 \Gamma_{0l} + \frac{N_0 \Gamma_{0u}}{\gamma_{ul} + \gamma_{u0}} \gamma_{ul}}{\gamma_{l0}}$$

$$= \frac{N_0 \left[\Gamma_{0l} + \left(\frac{\gamma_{ul}}{\gamma_{ul} + \gamma_{u0}} \right) \Gamma_{0u} \right]}{\gamma_{l0}} \qquad (9.19)$$

$$\frac{g_l}{g_u} \frac{N_u}{N_l} = \frac{\frac{N_0 \Gamma_{0u}}{\gamma_{ul} + \gamma_{u0}}}{N_0 \frac{\left[\Gamma_{0l} + \frac{\gamma_{ul}}{\gamma_{ul} + \gamma_{u0}} \Gamma_{0u} \right]}{\gamma_{l0}}}$$

$$\therefore \quad \frac{g_l N_u}{g_u N_l} = \frac{\gamma_{l0}}{(\gamma_{ul} + \gamma_{u0})} \left[\frac{\Gamma_{0u}}{\Gamma_{0l} + \frac{\gamma_{ul}}{\gamma_{ul} + \gamma_{u0}} \Gamma_{0u}} \right] \qquad (9.21)$$

4. Refer to the energy-level diagram, Figure 10-5(b), of the argon ion with the relevant energy levels for the 488.0-nm laser transition. Assuming that it operates as a three-level laser when considering only the single-ion energy levels, what would be the maximum ratio of excitation from the ground state to the lower laser level when compared to the excitation rate from the ground state to the upper laser level at threshold?

From (7.35) for inversion: $\frac{g_l N_u}{g_u N_l} > 1$

From Fig 10-5(b) $g_u = 2(5/2)+1 = 6$
$g_l = 2(3/2)+1 = 4$

$\therefore \frac{N_u}{N_l} > \frac{6}{4} = 1.5$

from (9.21) $$\frac{N_u}{N_l} = \frac{\Gamma_{ou} A_{lo}}{A_{ul}(\Gamma_{ol} + \Gamma_{ou}) + A_{uo}\Gamma_{ol}} > 1.5$$

or $$\frac{N_u}{N_l} \cong \frac{1}{(1 + \Gamma_{ol}/\Gamma_{ou})} \frac{A_{lo}}{A_{ul}} > 1.5$$

Hence $$\frac{\Gamma_{ol}}{\Gamma_{ou}} < \left[\frac{A_{lo}}{A_{ul}} \frac{1}{1.5} - 1\right]$$

but $A_{lo} = 23 \times 10^8/s + 4.5 \times 10^8/s = 27.5 \times 10^8/s$
+ $A_{ul} = 7.8 \times 10^7/s$

$$\therefore \frac{\Gamma_{ol}}{\Gamma_{ou}} < \frac{27.5 \times 10^8}{(7.8 \times 10^7)(1.5)} - 1 = \underline{22.5}$$

5. Given the rate equations (9.25), (9.26), (9.27), and (9.28), obtain the expressions for N_u, N_l, and Γ_{0i} for a four-level laser system given in (9.29), (9.30), and (9.33).

$$\frac{dN_l}{dt} = \gamma_{0l} N_0 - \gamma_{l0} N_l + \gamma_{ul} N_u = 0 \qquad (9.25)$$

$$\frac{dN_u}{dt} = -\gamma_{ul} N_u + \gamma_{iu} N_i = 0 \qquad (9.26)$$

$$\frac{dN_i}{dt} = \Gamma_{0i} N_0 - \gamma_{iu} N_i = 0 \qquad (9.27)$$

$$\text{but} \quad N_0 + N_l + N_u + N_i = N \qquad (9.28)$$

From (9.27) $\quad N_i = \dfrac{\Gamma_{0i} N_0}{\gamma_{iu}}$

Substitute into (9.26) $\quad N_u = \dfrac{\gamma_{iu} N_i}{\gamma_{ul}} = \dfrac{\gamma_{iu}}{\gamma_{ul}} \dfrac{\Gamma_{0i} N_0}{\gamma_{iu}}$

$$\text{or} \quad \underline{N_u = \frac{\Gamma_{0i} N_0}{\gamma_{ul}}}$$

from (9.25) $\quad N_l = \dfrac{\gamma_{0l} N_0 + \gamma_{ul} N_u}{\gamma_{l0}} = \dfrac{\gamma_{0l} N_0}{\gamma_{l0}} + \dfrac{\gamma_{ul}}{\gamma_{l0}} \dfrac{\Gamma_{0i} N_0}{\gamma_{ul}}$

$$= \frac{(\gamma_{0l} + \Gamma_{0i})}{\gamma_{l0}} N_0$$

$$\therefore \quad \frac{N_u}{N_l} = \frac{\Gamma_{0i}\, \gamma_{l0}}{\gamma_{ul}(\gamma_{0l} + \Gamma_{0i})} = \underline{\frac{\gamma_{l0}}{\gamma_{ul}} \frac{\Gamma_{0i}}{(\gamma_{0l} + \Gamma_{0i})} \overset{?}{>} 1}$$

$$\text{or} \quad \Gamma_{0i} > \frac{\gamma_{0l}\, \gamma_{ul}}{\gamma_{l0} - \gamma_{ul}} \qquad \text{but } \gamma_{l0} >> \gamma_{ul}$$

$$\therefore \quad \Gamma_{0i} > \frac{\gamma_{0l}}{\gamma_{l0}} \gamma_{ul} \qquad \text{but } \frac{\gamma_{0l}}{\gamma_{l0}} = e^{-\Delta E_{l0}/kT}$$

$$\therefore \quad \underline{\Gamma_{0i} > e^{-\Delta E_{l0}/kT} \gamma_{ul} \quad \text{or} \quad \Gamma_{0i} > e^{-\Delta E_{l0}/kT} A_{ul}}$$

6. Determine the single-pass gain of a 0.1-m–long Nd:YAG laser rod operating at 1.06 μm at room temperature. Assume the following:

(a) $A_{ul} = 4 \times 10^3\ \mathrm{s}^{-1}$, no significant collisional or phonon broadening occurs on that transition, and there exist no radiative decay routes from level u other than to level l;

(b) the pumping level i decays primarily to the upper laser level u, and the lower laser level l decays to the ground state 0, at a rate of $10^{12}\ \mathrm{s}^{-1}$;

(c) the lower laser level l is 0.27 eV above the ground state;

(d) the pumping rate to the intermediate level i is 100 times the minimum value given in (9.33); and

(e) the doping concentration of the Nd:YAG rod is $10^{26}\ \mathrm{m}^{-3}$.

$$\Gamma_{0i} = 100\, e^{-\Delta E_{l0}/kT} \gamma_{ul}$$

Assume $g_u = g_l$

$T \cong 300\mathrm{K}$

$$\Delta E_{l0} = 0.27\ \mathrm{eV} \qquad kT = \frac{1.38\times10^{-23}\ \mathrm{J/K}\ 300\mathrm{K}}{1.6\times10^{-19}\ \mathrm{J/eV}} = 2.59\times10^{-2}\ \mathrm{eV}$$

$$\gamma_{ul} = A_{ul} = 4\times10^3/\mathrm{s}$$

$$\therefore\ \Gamma_{0i} = 100\, e^{-\frac{0.27}{2.59\times10^{-2}}}\ 4\times10^3/\mathrm{s} = 11.88$$

From (9.29) $$N_u = \frac{\Gamma_{0i}}{\gamma_{ul}} N_0 = \frac{11.88}{4\times10^3}\ 10^{26}/\mathrm{m}^3 = 2.97\times10^{23}/\mathrm{m}^3$$

From (9.30) $$N_l = \frac{\gamma_{0l} + \Gamma_{0i}}{\gamma_{l0}} N_0 = \left[\frac{\gamma_{0l}}{\gamma_{l0}} + \frac{\Gamma_{0i}}{\gamma_{l0}}\right] N_0$$

$$= \left[e^{-\Delta E_{l0}/kT} + \frac{\Gamma_{0i}}{\gamma_{l0}}\right] N_0$$

$$= \left[e^{-\frac{0.27}{2.59\times10^{-2}}} + \frac{11.88}{10^{12}}\right] N_0$$

$$= 2.97\times10^{-5} N_0 = 2.97\times10^{21}$$

From Table 15-5 $\sigma_{ul}^H = 2.8\times10^{-23}\ \mathrm{m}^2$

Therefore $$g_0 = (N_u - N_l)\,\sigma_{ul}^H = (2.97\times10^{23} - 2.97\times10^{21})\,2.8\times10^{-23}\ \mathrm{m}^2$$

$$= 8.23/\mathrm{m}$$

$$I = I_0\, e^{8.23(.1)} = I_0\, e^{0.823} \quad \therefore\ \frac{I}{I_0} = e^{0.823} = 2.28$$

7. Beginning with the differential rate equations (9.35) and (9.36), solve for the expressions for N_u and N_l as a function of time as given in (9.37) and (9.38).

$$\frac{dN_u}{dt} = N_0 \Gamma_{0u} - N_u A_{ul} \qquad (9.35)$$

$$\frac{dN_l}{dt} = N_0 \Gamma_{0l} + N_u A_{ul} \qquad (9.36)$$

Assume N_0 = constant (a reasonable assumption)

Then assume a solution for $N_u \rightarrow N_u = C_1 (1 - e^{-A_{ul} t})$

C_1 = constant

Then $\frac{dN_u}{dt} = C_1 A_{ul} e^{-A_{ul} t}$

plug in (9.35) $C_1 A_{ul} \cancel{e^{-A_{ul} t}} = N_0 \Gamma_{0u} - C_1 A_{ul} (1 - \cancel{e^{-A_{ul} t}})$

$\therefore C_1 = \frac{N_0 \Gamma_{0u}}{A_{ul}}$ and $N_u = \frac{N_0 \Gamma_{0u}}{A_{ul}} (1 - e^{-A_{ul} t})$

plug in (9.36) $\frac{dN_l}{dt} = N_0 \Gamma_{0l} + \frac{N_0 \Gamma_{0u}}{\cancel{A_{ul}}} (1 - e^{-A_{ul} t}) \cancel{A_{ul}}$

$= N_0 \left[\Gamma_{0l} + \Gamma_{0u} (1 - e^{-A_{ul} t}) \right]$

Integrate: $N_l = N_0 \left[(\Gamma_{0l} + \Gamma_{0u}) t + \frac{\Gamma_{0u}}{A_{ul}} (1 - e^{-A_{ul} t}) \right]$

$= \frac{N_0 \Gamma_{0u}}{A_{ul}} \left[\left(\frac{\Gamma_{0l}}{\Gamma_{0u}} + 1 \right) A_{ul} t - (1 - e^{-A_{ul} t}) \right]$

$\therefore \frac{N_u}{N_l} = \frac{(1 - e^{-A_{ul} t})}{\left(\frac{\Gamma_{0l}}{\Gamma_{0u}} + 1 \right) A_{ul} t - (1 - e^{-A_{ul} t})}$ but $A_{ul} = \frac{1}{\tau_u}$

$\therefore \frac{N_u}{N_l} = \frac{(1 - e^{-t/\tau_u})}{\left(\frac{\Gamma_{0l}}{\Gamma_{0u}} + 1 \right) t/\tau_u - (1 - e^{-t/\tau_u})} \overset{?}{>} 1$

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8. Referring to the energy-level diagram of the lead vapor laser of Problem 8 in Chapter 4, graph the time dependence of the population inversion (in real time) of the $^3P_1^o \rightarrow {}^1D_2$ 722.9-nm laser transition, assuming that (a) the pumping flux to the upper laser level is twice that to the lower laser level and (b) the pumping is applied at a constant rate beginning at $t = 0$. Assume that radiation trapping prevents any decay to the three levels ($^3P_2$, $^3P_1$, $^3P_0$) below the lower laser level, and assume there is essentially no decay to the higher-lying $^1S_0$ level as shown in the figure.

$$\frac{\Gamma_{ou}}{\Gamma_{ol}} = 2 \quad \text{or} \quad \frac{\Gamma_{ol}}{\Gamma_{ou}} = 0.5 \qquad \Gamma_{ou} = C\tau$$

where τ = constant

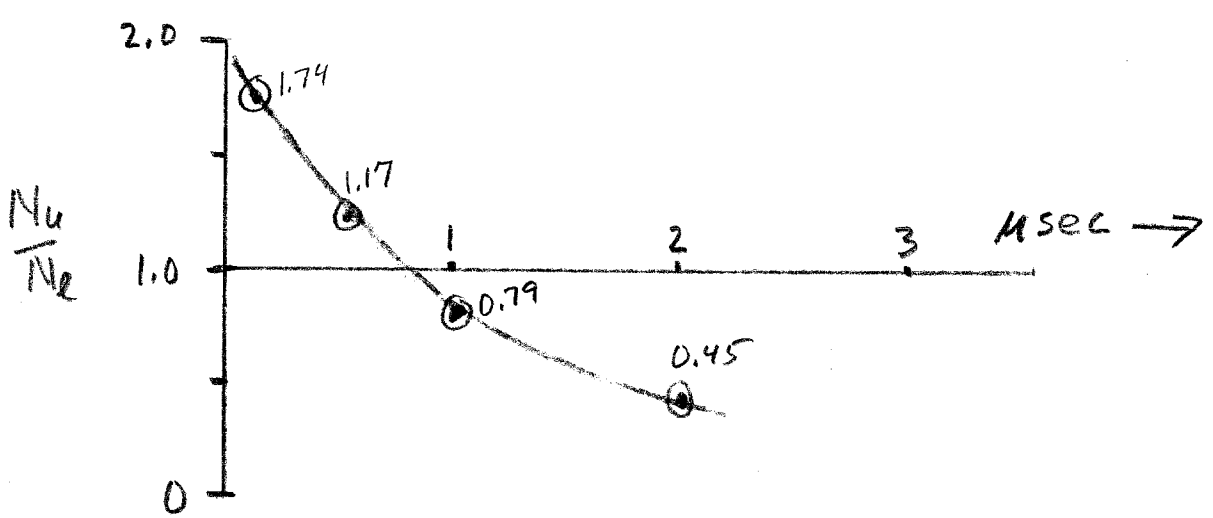
due to radiation trapping, $A_{3P_1^o \rightarrow {}^3P_2, {}^3P_1, {}^3P_0} = 0$

$$\therefore A_{3P_1^o} \cong A_{(3P_1^o \rightarrow {}^1D_2)} = 8.9 \times 10^5 / s$$

$$\text{Then } \tau_u = \frac{1}{A_{3P_1^o}} = \frac{1}{8.9 \times 10^5 / s} = 1.12 \times 10^{-6} s$$

$$\frac{N_u}{N_l} = \frac{1 - e^{-\tau / 1.12 \times 10^{-6} s}}{(1.5)\frac{\tau}{1.12 \times 10^{-6} s} - \left(1 - e^{-\tau / 1.12 \times 10^{-6} s}\right)}$$

$$\text{when } \tau = \tau_u \quad \frac{N_u}{N_l} = \frac{1 - e^{-1}}{1.5 - (1 - e^{-1})} = 0.423$$



9. At what electron density would radiation trapping begin to slow down the decay of the lower laser level of the He–Cd laser transition at 441.6 nm? Assume a plasma tube radius of 1 mm. Assume that the gas temperature is 350°C and that half of the ions in the discharge are Cd^+ ions while the other half are He^+ ions. Assume also that there are no other ions in the discharge and that charge neutrality exists: the total electron charge of the free electrons that provide the discharge current equals the total ion charge in the discharge. Use the energy-level diagram and the associated coefficients of Figure 9-7 for the relevant Cd^+ energy levels. *Hint:* The Cd^+ ion ground state is the state that causes radiation trapping on the decay of the lower laser level, and its population accounts for half of the free electrons that make up the electron density in the discharge (the other half are He^+ ions).

Radiation trapping becomes significant when

$$\sigma_{0\ell} N_0 b = 1.46 \qquad (9.47)$$

$$\sigma_{0\ell} = \frac{g_\ell}{g_0}\sigma_{\ell 0} = \frac{g_\ell}{g_0}\sqrt{\frac{\ln 2}{16\pi^3}}\,\frac{\lambda_{\ell 0}^2 A_{\ell 0}}{\Delta\nu_{\ell 0}^D}$$

$g_\ell = 2(3/2)+1 = 4$

$g_0 = 2(1/2)+1 = 2$

$T = 350+273 = 623\,K$

$b = 10^{-3}\,m$

$\lambda_{\ell 0} = 214.5\,nm$

$A_{\ell 0} = 2.8\times10^8\,/s$

$$\Delta\nu^D = 7.16\times10^{-7}\nu_0\sqrt{\frac{T}{M_N}}$$

$$= 7.16\times10^{-7}\,\frac{3\times10^8}{214.5\times10^{-9}}\sqrt{\frac{623}{114}} = 2.34\times10^9\,Hz$$

$$\therefore\ \sigma_{0\ell} = \frac{4}{2}\sqrt{\frac{\ln 2}{16\pi^3}}\,\frac{(214.5\times10^{-9})\,2.8\times10^8}{2.34\times10^9} = 4.12\times10^{-16}\,m^2$$

$$\sigma_{0\ell} N_0 b = 1.46$$

$$N_0 = \frac{1.46}{\sigma_{0\ell} b} = \frac{1.46}{4.12\times10^{-16}\,m^2\,10^{-3}\,m} = 3.54\times10^{18}\,/m^3$$

Total electron density $= 2N_0 = \underline{7.08\times10^{18}\,/m^3}$

10. At what population density of the argon ion ground state would radiation trapping begin to reduce the population inversion for the laser transition of Problem 4? Assume a discharge tube diameter of 2 mm and a gas temperature of 600 K.

$$\sigma_{0\ell} N_0 b = 1.46$$

$\lambda_{u\ell} = 488.0$ nm

$\lambda_{\ell 0} = 72.3$ nm

$b = 1$ mm

$M_N = 40$

$T = 600$ K

$g_\ell = 2(3/2)+1 = 4$

$g_0 = 2(3/2)+1 = 4$

$$\sigma_{0\ell} = \frac{g_\ell}{g_0}\sqrt{\frac{\ln 2}{16\pi^3}}\,\frac{\lambda_{\ell 0}^2 A_{\ell 0}}{\Delta\nu_D}$$

$$\Delta\nu_D = 7.16\times10^{-7}\,\nu_0\sqrt{\frac{T}{M_N}}$$

$$= 7.16\times10^{-7}\,\frac{3\times10^8}{488\times10^{-9}}\sqrt{\frac{600}{40}}$$

$$= 1.15\times10^{10}\ \text{Hz}$$

$A_{\ell 0} = 23\times10^8/\text{s}$ @ 72.3 nm

$$\sigma_{0\ell} = \left(\frac{4}{4}\right)\sqrt{\frac{\ln 2}{16\pi^3}}\,\frac{(72.3\times10^{-9})^2\,23\times10^8}{1.15\times10^{10}}$$

$$= 3.91\times10^{-17}\ \text{m}^2$$

$$N_0 = \frac{1.46}{\sigma_{0\ell}\, b} = \frac{1.46}{3.91\times10^{-17}\,10^{-3}}$$

$$= \underline{3.73\times10^{19}/\text{m}^3}$$

11. For the H-like $n = 3 \rightarrow n = 2$ transition of the B^{4+} ion, what is the maximum electron density at which a population inversion could occur for an electron and ion temperature of 300 eV? What would the gain be on that transition under those conditions, assuming that the pumping flux to the lower laser levels (other than radiative decay from the $n = 3$ level) is negligible? Assume that the population in the upper laser level is $10^{-5} n_e$. Note that the transition probabilities for H-like ions scale as Z^4 compared to hydrogen and that $A_{32} = 4.4 \times 10^7\ \mathrm{s}^{-1}$ for hydrogen.

$$n_e^{max} = \frac{0.13\sqrt{T_e}}{\lambda_{ul}^3}$$

T_e (K)
λ (nm) $Z = 5$ for B^{4+}

$$\lambda_{B^{4+}} = \frac{\lambda_H}{Z^2}$$

$$T = 300\ \mathrm{eV} = \frac{300}{8.62\times10^{-5}\ \mathrm{eV/K}} = 3.48\times10^6\ \mathrm{K}$$

$$\lambda_H(3\rightarrow2) = 656.3\ \mathrm{nm}$$

$$\lambda_{B^{4+}} = \frac{656.3}{(5)^2} = 26.25\ \mathrm{nm}$$

$$n_e^{max} = \frac{0.13\sqrt{3.48\times10^6}}{(26.25\times10^{-9})^3} = 1.34\times10^{25}/\mathrm{m}^3$$

<u>Estimate gain</u> Assume $N_u = 10^{-5} n_e^{max} = 1.34\times10^{20}/\mathrm{m}^3$

$$\sigma_{ul} = (1.74\times10^{-4})\,\lambda^3 A_{ul}\sqrt{\frac{M_N}{T}}$$

$M_N = 11$

$$A_{ul} = Z^4 A_{ul}^H = (5)^4\, 4.4\times10^7/\mathrm{s} = 2.75\times10^{10}/\mathrm{s}$$

$$\sigma_{ul} = (1.74\times10^{-4})(26.25\times10^{-9})^3\, 2.75\times10^{10}\sqrt{\frac{11}{3.48\times10^6}}$$

$$= 1.54\times10^{-19}\ \mathrm{m}^2$$

If we neglect N_l:

$$g = \sigma_{ul} N_u = 1.54\times10^{-19}\ \mathrm{m}^2\ 1.34\times10^{20}/\mathrm{m}^3 = \underline{20.6/\mathrm{m}}$$

(continued)

11 (cont.)

<u>If we include N_ℓ:</u>

$$N_u = \frac{N_0 \Gamma_{0u}}{\gamma_{u\ell} + \cancel{\gamma_{u0}}}$$

but $\gamma_{u\ell} \gg \gamma_{u0}$ for gas lasers

$$N_0 = \frac{n_e^{max}}{4} = \frac{1.3\times10^{25}/m^3}{4}$$

since for Be^{++} there are 4 electrons removed for every ground state ion.

$$\gamma_{u\ell} = A_{u\ell} = 2.75\times10^{10}$$

$$\gamma_{\ell 0} = A_{\ell 0} = Z^4 A_{\ell 0}^H$$

$$= 5^4\, 4.7\times10^8 = 2.94\times10^{11}$$

$$N_\ell = \frac{N_0\left[\cancel{\Gamma_{0\ell}} + \Gamma_{0u}\frac{\gamma_{u\ell}}{\gamma_{u\ell}+\cancel{\gamma_{u0}}}\right]}{\gamma_{\ell 0}} = \frac{N_0\Gamma_{0u}}{\gamma_{\ell 0}}$$

From above $\Gamma_{0u} = \dfrac{N_u A_{u\ell}}{N_0} = \dfrac{1.34\times10^{20}\, 2.75\times10^{10}}{1.3\times10^{25}/4} = 1.13\times10^6$

$$\therefore\ N_\ell = \frac{1.3\times10^{25}}{4}\,\frac{1.13\times10^6}{2.94\times10^{11}} = \underline{1.25\times10^{19}/m^3}$$

but $g_u = 18$
$g_\ell = 8$

$$\therefore\ g = \sigma_{u\ell}\left(N_u - \frac{g_u}{g_\ell}N_\ell\right)$$

$$= 1.54\times10^{-19}\left[1.34\times10^{20} - \frac{18}{8}\,1.25\times10^{19}\right]$$

$$= \underline{16.3/m}$$

12. For a discharge-pumped gas laser medium, assume that the electron pumping flux is twice as great to the upper laser level as to the lower laser level. Also assume that the decay rate from the lower laser level is twice as great as that from the upper laser level. The decay from the upper laser level goes only to the lower laser level and there is no other source of excitation to either the upper or lower laser levels than those just described. What is the ratio of N_u to N_l under these conditions? What would the value of the trapping factor F_{l0} (see eqn. 9.45) have to be in order to equalize the populations of the levels? Assume that the statistical weights of both laser levels are the same.

(a)
$$\frac{g_\ell}{g_u}\frac{N_u}{N_\ell} \cong \frac{g_\ell}{g_u}\frac{1}{\left(1+\Gamma_{o\ell}/\Gamma_{ou}\right)}\frac{A_{\ell o}}{A_{u\ell}}$$

but $g_\ell = g_u$ $\quad \frac{\Gamma_{o\ell}}{\Gamma_{ou}} = \frac{1}{2}$ $\quad \frac{A_{\ell o}}{A_{u\ell}} = 2$

$$\therefore \frac{N_u}{N_\ell} = \frac{1}{(1+1/2)}(2) = \frac{2}{1.5} = \underline{1.33}$$

(b)
$$1 = \frac{1}{(1+1/2)}F_{\ell o}(2)$$

$$\therefore F_{\ell o} = \frac{(1)(1.5)}{2} = \underline{0.75}$$