SOLUTION SET

Chapter 10

LASER PUMPING REQUIREMENTS AND TECHNIQUES

"LASER FUNDAMENTALS"

Second Edition

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1. If a laser-pumping light source emits a power of 10 W at a wavelength of 500 nm, and if we assume that 10% of the photons are absorbed by the laser gain material within the useful mode volume of the laser that is being pumped, then how many photons per second are absorbed in the material?

10 W @ 500 nm

1 W absorbed = 1 J/sec absorbed

$$h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,\text{Js}\;\;3\times10^{8}\,\text{m/s}}{500\times10^{-9}\,\text{m}}$$

$$= 3.98\times10^{-19}\,\text{J/photon}$$

$$1\,\frac{\text{J}}{\text{sec}}\;\frac{1}{3.98\times10^{-19}\,\text{J/photon}} = \underline{2.51\times10^{8}\,\text{photons/s}}$$

2. In Problem 1, how many upper laser level species would accumulate within an upper laser level lifetime of 5 ns (dye laser), 3.8 μs (titanium:sapphire laser), 230 μs (Nd:YAG laser), and 3 ms (ruby laser)? Assume that one upper laser level species is produced for every pump photon absorbed within the material.

		upper laser level species
dye	(a) 2.51×10^{18} photons/s $\times$ 5×10^{-9} s	$= 1.26\times10^{10}$
$Ti:Al_2O_3$	(b) 2.51×10^{18} photons/s $\times$ 3.8×10^{-6} s	$= 9.54\times10^{12}$
Nd:YAG	(c) 2.51×10^{18} photons/s $\times$ 230×10^{-6} s	$= 5.77\times10^{14}$
Ruby	(d) 2.51×10^{18} photons/s $\times$ 3×10^{-3} s	$= 7.35\times10^{15}$

3. The flux from the sun arriving at the surface of the earth is approximately 1 kW/m^2 on average during the daytime. If 10% of that flux falls within the pumping band of a Nd:YAG laser crystal, how big would a collecting lens have to be (in diameter) to collect enough power to pump the Nd:YAG laser to reach laser threshold if the laser crystal is 0.1 m long and 6 mm in diameter? Assume that all of the sunlight passing through the lens is concentrated within the laser rod such that all of the flux within the pump absorption bandwidth is absorbed and converted to upper laser level species. Assume that the flux accumulates for a duration of the lifetime of the upper laser level and that the average photon energy of the solar flux within the absorption pumping band of the Nd:YAG rod is 2 eV. Also assume that both of the laser mirrors have a reflectivity of 95%.

Solar flux $\cong$ 1 kW/m^2 10% in pumping band of Nd:YAG

$R_1 = R_2 = 0.95$ $L = 0.1$ m diameter = 6 mm

Threshold gain $\sigma_{ul} N_u L = \sigma_{ul} N_j \Gamma_{ju} \tau_u L = \frac{1}{2} \ln \frac{1}{R_1 R_2}$

For Nd:YAG $\sigma_{ul} = 2.8 \times 10^{-23}\ m^2$ $\tau_u = 230\ \mu s$

Flux into u $N_j \Gamma_{ju} = \dfrac{\frac{1}{2} \ln \frac{1}{R_1 R_2}}{\sigma_{ul} \tau_u L} = \dfrac{\frac{1}{2} \ln \frac{1}{(0.95)^2}}{2.8 \times 10^{-23}\ 230 \times 10^{-6}\ (0.1)}$

$= 7.96 \times 10^{25} / m^3 s$

Photons/sec $= 7.96 \times 10^{25} \frac{\text{photons}}{m^3 s} (0.1\ m)\, \pi\, (3 \times 10^{-3})^2 m^2 = 2.25 \times 10^{20}$ photons/s

2.25×10^{20} photons/s $\frac{2\ eV}{\text{photon}} \frac{1.6 \times 10^{-19} J}{eV} = 72.0\ W$

For 10% pumping efficiency, would require $\frac{72.0}{0.10} = 720\ W$

What diameter lens would provide 720 W ?

Area $= \dfrac{720\ W}{1000\ W/m^2} = 0.72\ m^2 = \pi \dfrac{d^2}{4}$

$d = \sqrt{\dfrac{4(.72)}{\pi}} = \underline{.957\ m = 95.7\ cm}$

4. A 5-W argon ion laser operating at 514.5 nm is used as a pumping beam for a Rh6G dye laser. The dye gain medium consists of a rapidly flowing Rh6G dye jet stream of thickness 0.5 mm located within a cavity similar to that shown in Figure 10-16(g). The dye concentration is of sufficient density such that 50% of the pump beam is absorbed in the dye. It is desirable that the pump beam spot size be the same as the dye laser spot size in order to provide efficient absorption of the energy. What spot size (diameter) would the pump laser and the dye laser need within the dye jet stream in order to produce a single-pass exponential gain of unity (i.e., $\sigma_{ul}^{H} N_u L = 1$)? Assume the upper laser level lifetime of the dye is 5 ns and every pump photon that is absorbed is converted to an upper laser level species within the lifetime of that level. Assume that the lower laser level population is negligible because it rapidly decays to the bottom of the singlet ground state, and assume that $\sigma_{ul} = 2.5 \times 10^{-20}$ m^2. If the spot size were increased to 1 mm *diameter,* by what factor would the gain be reduced?

$\sigma_{ul} N_u L = 1$ From Fig 7-10 $\sigma_{ul} = 2.5 \times 10^{-20}\ m^2$

$N_u L = \frac{1}{\sigma_{ul}} = \frac{1}{2.5 \times 10^{-20}\ m^2} = 4 \times 10^{19}/m^2$

Every absorbed photon → upper laser level species

$\therefore$ pump photon flux $= \underset{\text{50\% efficient}}{2} \frac{N_u L}{\tau_u} = \frac{(2)\,4 \times 10^{19}/m^2}{5 \times 10^{-9}\ s} = 1.6 \times 10^{28} \frac{\text{photons}}{m^2 s}$

photon energy $= \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34}\ Js\ 3 \times 10^{8}\ m/s}{(514.5 \times 10^{-9}\ m)} = 3.85 \times 10^{-19} \frac{J}{\text{photon}}$

$I = 1.6 \times 10^{28} \frac{\text{photons}}{m^2 s}\ 3.85 \times 10^{-19} \frac{J}{\text{photon}} = 6.16 \times 10^{9}\ W/m^2$

$\text{Area} = \frac{P}{I} = \frac{5\ W}{6.16 \times 10^{9}\ W/m^2} = 8.12 \times 10^{-10}\ m^2 = \pi \left(\frac{d}{2}\right)^2$

$\therefore\ d = \sqrt{\frac{4\ 8.12 \times 10^{-10}}{\pi}} = 3.22 \times 10^{-5}\ m = \underline{32.2\ \mu m}$

If spot size = 1 mm diameter

$I = \frac{P}{\text{Area}} = \frac{5\ W}{\pi (0.5 \times 10^{-3})^2} = 6.37 \times 10^{6}\ W/m^2$

photon flux $= \frac{6.37 \times 10^{6}\ W/m^2}{3.85 \times 10^{-19}\ J/ph.} = 1.65 \times 10^{25}$ photons/m^{2}s

$N_u L = \frac{\text{pump photon flux} \cdot \tau_u}{2} = \frac{1.65 \times 10^{25}\ 5 \times 10^{-9}}{2} = 4.13 \times 10^{16}/m^2$

$\therefore\ \sigma_{ul} N_u L = 2.5 \times 10^{-20}\ m^2\ 4.13 \times 10^{16}/m^2 = \underline{1.03 \times 10^{-3}}$ Not good!

Gain reduced by $\frac{\pi \left(\frac{0.032}{2}\right)^2}{\pi \left(\frac{1.0}{2}\right)^2} = 1.02 \times 10^{-3}$ A significant reduction!

5. An electrical current is applied to a copper vapor laser discharge tube filled with Cu atoms at a density of 10^{23} atoms per cubic meter. The electron temperature obtained within the discharge is approximately 15,000 K. Assume that the velocity-averaged electron excitation cross section from the Cu atom ground state to the upper laser level is $\sigma_{0u}^{e} \cong 10^{-22}$ m^2. At what electron density would the pumping flux be sufficient to produce a single-pass gain coefficient of 10 m^{-1}? Assume that the lifetime of the upper laser level is 5×10^{-7} s. *Hint:* Determine the average velocity of the electrons for the given temperature.

Copper vapor laser $\quad T_e = 15{,}000$ K

$\sigma_{0u}^{e} \cong 10^{-22}\ m^2$

Find n_e to produce gain coefficient of $10/m$

$\sigma_{ue} N_u = \sigma_{ue} N_j \Gamma_{ju} \tau_u = 10/m$ (where $j = 0$)

(10.7) $\Gamma_{0u} = n_e \bar{v}_e \bar{\sigma}_{0u}^{e}$ $\quad$ Fig(7-10) $\sigma_{ue} = 8.6 \times 10^{-18}\ m^2$

$\therefore \sigma_{ue} N_0 n_e \bar{v}_e \bar{\sigma}_{0u}^{e} \tau_u = 10/m$

$$n_e = \frac{10/m}{\sigma_{ue} N_0 \bar{v}_e \bar{\sigma}_{0u}^{e} \tau_u}$$

From(10.37) $\quad \bar{v}_e = \sqrt{\frac{8kT_e}{m_e \pi}} = \sqrt{\frac{8(1.38 \times 10^{-23})15{,}000}{9.11 \times 10^{-31}\ \pi}}$

$= 7.6 \times 10^5\ m/s$

$$\therefore n_e = \frac{10/m}{(8.6 \times 10^{-18} m^2)(10^{23}/m^3)(7.6 \times 10^5\ m/s)(10^{-22} m^2)(5 \times 10^{-7} s)}$$

$= \underline{3.06 \times 10^{17}/m^3}$

6. A flashlamp is available that radiates a power of 25 W within the pumping band of a Nd:YAG laser at an average wavelength of 700 nm. Approximately 15% of that power is collected by the laser rod and absorbed *uniformly* within the rod by the use of a multi-elliptical cavity. The rod is doped with Nd ions such that the absorption length of the pump radiation is equal to the diameter of the rod. For a 0.1-m–long, 6-mm–diameter rod, what is the single-pass gain obtained with this pumping flux? The absorption cross section at 700 nm (to level q) is equal to the stimulated emission cross section from level u to level l. Assume that $A_{ul} = \gamma_u = 4.2 \times 10^3\ \mathrm{s}^{-1}$ and $\gamma_{l0} = \gamma_{qu} \approx 10^{12}\ \mathrm{s}^{-1}$ and that the laser operates at room temperature.

$L = 0.1\ \mathrm{m}$ 6 mm diameter rod

$\sigma_{ul} = 2.8 \times 10^{-23}\ \mathrm{m}^2$

$\tau_u = 230\ \mu\mathrm{s}$

photons absorbed

$$= \frac{25\ \mathrm{W}\ (.15)}{h\nu} = \frac{25\ \mathrm{J/s}\ (.15)\ 700 \times 10^{-9}\ \mathrm{m}}{(6.62 \times 10^{-34}\ \mathrm{Js})(3 \times 10^8\ \mathrm{m/s})} = 1.32 \times 10^{19}\ \frac{\text{photons}}{\mathrm{s}}$$

photons absorbed in one lifetime

$$= \left(1.32 \times 10^{19}\ \frac{\text{photons}}{\mathrm{s}}\right)(230 \times 10^{-6}\ \mathrm{s}) = 3.04 \times 10^{15}\ \text{photons}$$

Assume each photon excites one upper state species

Then $$N_u = \frac{3.04 \times 10^{15}\ \text{photons}}{\underbrace{(0.1\ \mathrm{m})\ \pi\ (3 \times 10^{-3}\ \mathrm{m})^2}_{\text{volume}}} = 1.08 \times 10^{21}/\mathrm{m}^3$$

Single pass gain $= \sigma_{ul} N_u L$

$$= 2.8 \times 10^{-23}\ \mathrm{m}^2 (1.07 \times 10^{21}/\mathrm{m}^3)(0.1\ \mathrm{m})$$

$$= \underline{3.0 \times 10^{-3}}$$ not much!

7. What pump intensity at 514.5 nm will be required to pump a Ti:Al_2O_3 laser rod optically to achieve a gain coefficient of 3/m if the rod is doped to a concentration of 10^{25} per cubic meter and the cross section associated with absorption of the pump flux at the pump wavelength exceeds the stimulated emission cross section of the laser by a factor of 3? Assume that the stimulated emission cross section of Ti:Al_2O_3 at the operating laser wavelength is 3.5×10^{-23} m^2.

$\lambda_p = 514.5\ nm \qquad g_0 = 3/m \qquad N_0 = 10^{25}/m^3$

$\sigma_{ul} = 3.5 \times 10^{-23}\ m^2 \qquad \sigma_{0u} = 3\,\sigma_{ul} \qquad \tau_u = 3.8\ \mu s$

$A_{ul} = \frac{1}{\tau_u} = 2.6 \times 10^5/s$

$$I = \frac{2\left(h\frac{c}{\lambda_p}\right)A_{ul}}{\pi\,\sigma_{0u}\,N_0\,\sigma_{ul}}\,g_0$$

$$= \frac{2\,(6.626\times10^{-34})\left(\frac{3\times10^8}{514.5\times10^{-9}}\right)2.6\times10^5\,(3)}{\pi\,3\,(3.5\times10^{-23})(10^{25})(3.5\times10^{-23})}$$

$$= \underline{5.22\times10^6\ W/m^2}$$

8. A 40-W cw argon ion laser operating at 514.5 nm is used to end pump a Rh6G dye jet stream of 500-μm thickness (effective gain length). The pump radiation is concentrated on the jet in a 0.1-mm–diameter circular spot. What molar concentration (moles/liter) of the dye in the solvent will produce a single-pass gain of 1% at 577 nm? Assume that the profile of the absorption cross section for the dye has the wavelength dependence shown in Figure 5-10. The peak absorption cross section is $\sigma_{abs} = 3.8 \times 10^{-20}$ m^2. The molecular weight of Rh6G is 479. The stimulated emission cross section is given in Figure 7-10. The radiative lifetime was given in Problem 4. Assume that the population in the lower laser level is negligible. *Hint:* The molecular weight in grams of a substance will provide Avogodro's number of molecules (6.023×10^{23} molecules) of that substance. If that amount is dissolved in one liter of solvent (10^{-3} m^3), it will provide 6.023×10^{26} dye molecules per cubic meter.

CHANGE THE PROBLEM TO 250mW INSTEAD OF 40W

From Fig. 5-10 absorption cross section at 514.5nm is

$$\sigma_{ol} = (0.94)\ 3.8\times10^{-20}\,m^2 \cong 3.6\times10^{-20}\,m^2$$

Area of absorption jet is $\pi\left(\frac{10^{-4}}{2}\right)^2 = 7.85\times10^{-9}\,m^2$

Volume of gain region is $7.85\times10^{-9}\,m^2\ 500\times10^{-6}\,m = 3.93\times10^{-12}\,m^3$

Stimulated emission cross section of dye is

$$\sigma_{ul} = 2.5\times10^{-20}\,m^2 \text{ (from Fig. 7-10)}$$

Single pass gain of 1% indicates that:

$$\frac{I}{I_0} = e^{\sigma\Delta N L} = 1.01 \Rightarrow \sigma\Delta N L = 0.01$$

Assume N_l is negligible $\Rightarrow \Delta N \cong N_u$

Then $\sigma_{ul} N_u L = 0.01 \qquad L = 500\times10^{-6}\,m$

$$\therefore N_u = \frac{0.01}{2.5\times10^{-20}\,m^2\ 500\times10^{-6}\,m} = 8\times10^{20}/m^3$$

$\therefore$ # of molecules in level u at any given time is:

$$8\times10^{20}/m^3\ 3.93\times10^{-12}\,m^3 = 3.14\times10^{9} \text{ molecules}$$

$\therefore$ We want 3.14×10^9 pump photons absorbed every 5 ns

250 mW = 0.25 W

each pump photon has energy of

$$h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\ 3\times10^{8}}{514.5\times10^{-9}} = 3.87\times10^{-19}\,J$$

$$\therefore \frac{.25\ J/s}{3.87\times10^{-19}\,J} = 6.46\times10^{17} \text{ photons/s}$$

(continued)

8 (cont.)

$$6.46\times10^{17}\ \frac{\text{photons}}{\text{s}}\ 5\times10^{-9}\,\text{s} = 3.23\times10^{9}\ \text{photons}$$

If these were all absorbed they could produce 3.23×10^{9} molecules in level u during one radiative lifetime of the upper laser level.

To absorb the pump beam within the depth of 500×10^{-6} m we would have

$$e^{-\sigma_{0u} N_0 L} = e^{-1}$$

or $\sigma_{0u} N_0 L = 1$ or $N_0 = \dfrac{1}{\sigma_{0u} L}$

but $\sigma_{0u} = 3.6\times10^{-20}\,\text{m}^2$ and $L = 500\times10^{-6}$ m

$$\therefore\ N_0 = 5.56\times10^{22}/\text{m}^3$$

for 1 molar solution we would have 6×10^{26} dye molecules/m^3

$\therefore$ the molar concentration is

$$\frac{5.56\times10^{22}/\text{m}^3}{6\times10^{26}/\text{m}^3} = \underline{9.27\times10^{-5} \cong 10^{-4}}$$

9. Assume the He–Cd upper laser level is populated by collisions with helium metastable atoms in a discharge tube. Assume there are $10^{18}/\text{m}^3$ He triplet metastable atoms in the discharge and that they are the primary source of excitation of the He–Cd laser; assume the Cd density in the discharge is $5 \times 10^{20}/\text{m}^3$. For a gas discharge temperature of 540 K, what must the population be in the upper laser level, and what would be the single-pass gain at 441.6 nm for a 0.3-m–long gain region? Assume Doppler broadening and that there is a single even isotope of Cd^{114} in the laser tube, so that there is no isotope broadening. Assume that the Penning ionization cross section from He metastables to Cd neutral ground-state atoms that populates the upper laser level is $\sigma_P = 10^{-19}\ \text{m}^2$. Assume also that there is no collisional depopulation of the upper laser level u and that radiative decay occurs from level u only to level l (i.e., there are no other decay pathways).

$N_{He}^T = 10^{18}/m^3 \quad N_{Cd} = 5\times10^{20}/m^3 \quad T = 540\ K$

$\lambda_{ul} = 441.6\ nm \quad A_{ul} = 1.4\times10^6/s \quad L = 0.3\ m$

$\sigma_P = 10^{-19}\ m^2$

Need to find $\sigma_{ul}^D N_u L = ?$

$$\Delta\nu_D = 7.16\times10^{-7}\frac{c}{\lambda_{ul}}\sqrt{\frac{T}{M_N}}$$

$$= 7.16\times10^{-7}\frac{3\times10^8}{441.6\times10^{-9}}\sqrt{\frac{540}{114}} = 1.06\times10^9\ Hz$$

$$N_u = N_{He}^T V_{He}\sigma_P N_{Cd}\tau_u \qquad \tau_u = \frac{1}{A_{ul}}$$

$$V_{He} = \sqrt{\frac{8kT}{M_{He}\pi}} = \sqrt{\frac{8\ 1.38\times10^{-23}\ 540}{4\ 1.67\times10^{-27}\ \pi}} = 1.69\times10^3\ m/s$$

$$\sigma_{ul}^D = \sqrt{\frac{\ln 2}{16\pi^3}}\ \frac{(441.6\times10^{-9})^2\ 1.4\times10^6}{1.06\times10^9} = 9.62\times10^{-18}\ m^2$$

$$\text{gain} = g_0 L = \sigma_{ul}^D N_u L = (9.62\times10^{-18})(6.04\times10^{16})(0.3)$$

$$= 0.174$$

$e^{.174} = \underline{1.19 \text{ per pass or } 19\% \text{ increase/pass}}$

10. An argon ion laser operates at a gas temperature of 1,200 K and pressure of 0.1 Torr and an electron temperature of 80,000 K. Assume the laser discharge has a length of 0.5 m and a bore diameter of 2 mm. Also assume that both mirrors have a transmission at the laser wavelength (488.0 nm) of 1% and that there are no other losses within the cavity. What must the electron collisional excitation cross section be in order for the laser to operate at threshold with a discharge current of 10 A? Assume the electron drift velocity is 10^5 m/s. Assume (i) that the gas is completely ionized and composed of singly ionized argon ions and electrons and (ii) that excitation of the upper laser level occurs by electron collisions with the argon ion ground state. An energy-level diagram of the relevant argon ion laser levels is shown in Figure 10-5(b).

Discharge current = 10 A $\quad L = 0.5\,m \quad T_e = 80{,}000\,K$

$P = 0.1$ Torr $\quad \bar{V}_D = 10^5\,m/s$

(10.38) $\sigma_{ue} N_u L = \frac{1}{2} \ln\left(\frac{1}{R^2}\right) = 1 \times 10^{-2}$

from Fig 7-10 for Ar^+, $\sigma_{ue} = 2.5 \times 10^{-16}\,m^2$

$$N_u = \frac{1 \times 10^{-2}}{(0.5\,m)(2.5 \times 10^{-16}\,m^2)} = 8 \times 10^{13}/m^3$$

But $N_u = N_0 \Gamma_{0u} \tau_u = N_0 (n_e \bar{V}_e \bar{\sigma}^e_{0u}) \tau_u$

$$\therefore \bar{\sigma}^e_{0u} = \frac{N_u}{N_0 n_e \bar{V}_e \tau_u}$$

$$\bar{V}_e = \sqrt{\frac{8kT_e}{m_e \pi}} = \sqrt{\frac{8(1.38 \times 10^{-23}\,J/K)\,80{,}000K}{9.11 \times 10^{-31}\,kg\,\pi}} = 1.75 \times 10^6\,m/s$$

From ideal gas law:

$$N_0 = \frac{P}{kT} = \frac{0.1\,Torr}{(1.38 \times 10^{-23}\,J/K)(1200K)} \left(\frac{1\,atm}{760\,Torr}\right)\left(\frac{101325\,Pa}{1\,atm}\right)\left(\frac{1\,N/m^2}{1\,Pa}\right)$$

$$= 8.05 \times 10^{20}/m^3$$

From (10.34) $j_e = n_e e \bar{V}_D$

$$\therefore n_e = \frac{j_e}{e \bar{V}_D} = \frac{I(amp)/area}{e \bar{V}_D} = \frac{10A/\pi(1mm)^2}{1.6 \times 10^{-19}C\ 10^5\,m/s} = 2 \times 10^{20}/m^3$$

$$\therefore \bar{\sigma}^e_{0u} = \frac{8 \times 10^{13}/m^3}{(8.05 \times 10^{20}/m^3)(2 \times 10^{20}/m^3)(1.76 \times 10^6\,m/s)(1.3 \times 10^{-8}\,s)}$$

$$= \underline{2.17 \times 10^{-26}/m^2}$$

11. A 0.1-m–long, 8-mm–diameter Nd:YAG laser rod is pumped with two linear flashlamps installed in a double elliptical pumping cavity. Each of the lamps provides a useful pumping power of 5 kW within the pump absorption band of the laser rod, and the lamps are operated for a duration of 200 μs. The operating temperature of the laser rod is 100°C and the index of refraction of the rod is 1.82. The rod is doped to a concentration of $3 \times 10^{26}/\text{m}^3$. The average pumping wavelength of the flashlamps is 500 nm. Assume that 70% of the pumping flux is absorbed uniformly by the laser rod and that each of those absorbed photons produces an upper laser level species. (a) What would be the single-pass small-signal gain (I/I_0) produced by this pumping arrangement? (b) By what percentage would the pump power have to be increased in order to double the single-pass gain? *Note:* For the conditions given, you must consider the populations of both the upper and lower laser levels.

$$\#\text{ pump photons absorbed} = \frac{(.7)(2)5\times10^3\,\text{J/s}\;200\times10^{-6}\,\text{s}}{2.48\,\text{eV/photon}\;1.6\times10^{-19}\,\text{J/eV}} = 3.52\times10^{18}\text{ photons}$$

$$\text{Volume of rod} = (0.1)\,\pi\,(0.004)^2 = 5.03\times10^{-6}\,\text{m}^3$$

(a) $$N_u = \frac{3.52\times10^{18}}{5.03\times10^{-6}\,\text{m}^3} = 7.0\times10^{23}/\text{m}^3 \qquad T = 373\,\text{K}$$

$$N_\ell \cong 3\times10^{26}/\text{m}^3\, e^{-\frac{0.262\,\text{eV}\;1.6\times10^{-19}\,\text{J/eV}}{1.38\times10^{-23}\;373}}$$

$$= 3\times10^{26}/\text{m}^3\;2.90\times10^{-4} = 8.71\times10^{22}/\text{m}^3$$

$$\sigma_{u\ell}(N_u - N_\ell)L = 2.8\times10^{-23}\,\text{m}^2(7 - .871)\times10^{23}/\text{m}^3\,(0.1\,\text{m})$$

$$= 1.716$$

$$I/I_0 = e^{1.716} = \underline{5.56}$$

double

(b) $$\frac{I}{I_0} = 2\times5.56 = 11.12 \qquad \sigma\Delta N L = 2.41$$

$$N_u - N_\ell = \frac{2.41}{\sigma_{u\ell}L} = 8.61\times10^{23}/\text{m}^3$$

$\therefore$ since N_ℓ is the same, $N_u = 9.48\times10^{23}/\text{m}^3$

$\therefore$ Pump ratio = N_u ratio = $\dfrac{9.48\times10^{23}}{7.0\times10^{23}} = \underline{1.35}$

<u>35% increase</u>

12. A Rh6G organic dye laser operating at 580 nm is end pumped (with an argon ion laser at 488 nm) in a thin jet stream gain region of 0.5-mm thickness to produce a single-pass gain of 10%. The laser is operated in a cavity with two mirrors, one with 100% reflectivity and the other with 95% reflectivity, and no other scattering losses exist within the cavity. Assume that part of the population in the singlet upper laser level decays to the triplet T_1 state. If the triplet absorption cross section at the laser wavelength (580 nm) has the same value as that of the peak pump absorption cross section at 488 nm (3.8×10^{-20} m^2), then what population density in the triplet state would just begin to stop the laser from operating? The stimulated emission cross section at the laser wavelength (580 nm) is 2.5×10^{-20} m^2.

$$\frac{I}{I_0} = 1.1 = e^{g_0 L} \Rightarrow g_0 L = 0.095$$

$$g_0 = \frac{0.095}{0.0005\,m} = 190.0/m$$

$$g_0 = \frac{1}{2(0.0005\,m)} \ln \frac{1}{(1.0)(0.95)} + \alpha = 190$$

$$\therefore \alpha = 190 - 51.29 = 138.71 = \sigma_T N_T$$

$$\therefore N_T = \frac{138.71}{3.8 \times 10^{-20}\,m^2} = \underline{3.65 \times 10^{21}/m^3}$$

13. A ruby laser operating at 694 nm is flashlamp pumped in the pumping band at 550 nm to the point that a population inversion is just barely achieved (i.e., at threshold). What would be the value of the useful pumping flux emitted from the lamp for those conditions, in photons/m^3-s, if 50% is absorbed within the laser rod? The rod is 0.1 m long with a 10-mm diameter and is doped to a Cr concentration of $10^{26}/m^3$. Assume there are cavity losses of 10% including the mirrors.

From (9.15) at Threshold $\Gamma_{\ell i} = A_{u\ell}$

$\therefore \Gamma_{\ell i} = 333/s$

At Threshold, half The population is in The lower laser level (ground state) and half is in The upper laser level.

$\therefore N_\ell \Gamma_{\ell i} = \frac{10^{26}/m^3}{2} 333/s = 1.67 \times 10^{28}/m^3 s$

Since only 50% is absorbed, Then useful pumping flux $= 2 \times 1.67 \times 10^{28}/m^3 s$

$= 3.33 \times 10^{28} \frac{\text{photons}}{m^3 s}$

This assumes That every photon absorbed creates an upper laser level.

14. What would be the necessary electron concentration in an InP triple quantum-well semiconductor laser operating at a current of 1,000 A/cm^2 if the quantum wells are each 10 nm thick? Assume a recombination time of 2×10^{-9} s and a diffusion coefficient of 2×10^{-3} m^2/s.

$$j = n_c e V_D$$

diffusion length $d \doteq (D_c \tau_R)^{1/2}$

$$D_c = 2 \times 10^{-3}\ m^2/s \qquad \tau_R = 2 \times 10^{-9}\ s$$

$$\therefore d = \left[(2 \times 10^{-3}\ m^2/s)(2 \times 10^{-9}\ s)\right]^{1/2} = 2 \times 10^{-6}\ m$$

but the quantum well is only $10\ nm = 10^{-8}\ m$

$$2 \times 10^{-6}\ m \gg 10^{-8}\ m \quad \therefore d = 10^{-8}\ m$$

$$\therefore V_D = \frac{d}{\tau_R} = \frac{10^{-8}\ m}{2 \times 10^{-9}\ s} = 5\ m/s$$

$$j = 1000\ A/cm^2 = 10^7\ A/m^2$$

$$\therefore n_c = \frac{j}{e V_D} = \frac{10^7\ A/m^2}{1.6 \times 10^{-19}\ C\ 5\ m/s}$$

$$= \underline{1.25 \times 10^{25}\ /m^3}$$