SOLUTION SET

Chapter 13

SPECIAL LASER CAVITIES AND CAVITY EFFECTS

"LASER FUNDAMENTALS"

Second Edition

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1. In designing an unstable resonator for a 248-nm KrF excimer laser, for practical considerations it is desirable to have a relatively long gain medium such that the separation between mirrors is 0.5 m. For a magnification factor of 5, what would be the radii of curvature of both the output mirror and the rear mirror? If the laser cavity were to be designed for minimum diffraction losses, what would be the diameter of the output mirror?

separation between mirrors is 0.5m

$$M = \frac{R_r}{R_0} = 5$$

$$R_r - R_0 = 2d$$

$$5R_0 - R_0 = 2d$$

$$R_0 = \frac{2d}{4} = \frac{d}{2} = \underline{0.25\,m}$$

$$R_r = 1\,m + 0.25\,m = \underline{1.25\,m}$$

$$N_{eq} = \left[\frac{(M-1)}{2M^2}\right]\left(\frac{a^2}{\lambda d}\right) = \left(\frac{5-1}{2\cdot 5^2}\right)\frac{a^2}{\lambda d} = 0.5$$

$$a^2 = (0.5)\lambda d\left(\frac{50}{4}\right) = (0.5)(248\times10^{-9})(0.5)\frac{50}{4}$$

$$a = 8.8\times10^{-4} = 0.88\,mm$$

$$\text{diameter} = 2\times 0.88 = \underline{1.76\,mm}$$

2. Beginning with (13.4), the constraint for a positive-branch confocal resonator that $R_r - R_o = 2d$, obtain (13.10) and (13.11).

$$R_r - R_o = 2d$$

but R_o has negative curvature so we replace R_o with $-R_o$ to obtain

$$R_r + R_o = 2d \quad \text{or} \quad \frac{R_r}{2} + \frac{R_o}{2} = d$$

$$g_1 = 1 - \frac{d}{R_1} \qquad g_2 = 1 - \frac{d}{R_2}$$

$$g_1 = 1 - \frac{d}{R_r} \qquad g_2 = 1 + \frac{d}{R_o}$$

$$g_1 = 1 - \frac{(R_r + R_o)}{2R_r} \qquad g_2 = 1 + \frac{(R_r + R_o)}{2R_o}$$

$$g_1 = 1 - \frac{1}{2} + \frac{R_o}{2R_r} \qquad g_2 = 1 + \frac{1}{2} + \frac{R_r}{2R_o}$$

$$g_1 = \frac{1}{2}\left(1 + \frac{R_o}{R_r}\right) \qquad g_2 = \frac{1}{2}\left(3 + \frac{R_r}{R_o}\right)$$

$$g_1 g_2 = \frac{1}{4}\left(3 + \frac{3R_o}{R_r} + \frac{R_r}{R_o} + \frac{R_o R_r}{R_r R_o}\right)$$

$$= \frac{1}{4}\left(4 + \frac{3R_o}{R_r} + \frac{R_r}{R_o}\right)$$

$$g_1 + g_2 = \frac{1}{2}\left(1 + \frac{R_o}{R_r} + 3 + \frac{R_r}{R_o}\right) = \frac{1}{2}\left(4 + \frac{R_o}{R_r} + \frac{R_r}{R_o}\right)$$

$$\therefore 2g_1 g_2 = g_1 + g_2$$

3. A 0.1-m–long Nd:YAG laser rod is installed in a 0.4-m–long laser cavity of a Q-switched laser. Determine the approximate value for t_s and show that it is significantly shorter than τ_u, the upper laser level lifetime.

$L = 0.1\,m$ (Nd:YAG) $\quad d = 0.4\,m \quad n_c = n_{air} \cong 1$

$n_L = 1.82 \qquad \tau_u = 230\,\mu s$

$$g_{th} m L \cong 12$$

where mL is the effective gain length

$$\therefore \; m \cong \frac{12}{g_{th} L} = \frac{12}{10\,m^{-1}\,0.1\,m} \cong 12$$

The minimum gain duration t_s for the beam to reach I_{sat} is:

$$t_s = \frac{m[n_c(d-L) + n_L L]}{c} \cong \frac{12[1\cdot(0.4-0.1) + 1.82(0.1)]}{3\times10^8}$$

$$\cong \underline{1.9\times10^{-8}\,s = 19\,ns \ll 230\,\mu s}$$

4. A 0.1-m–long ruby laser rod has the mirrors coated on the ends of the rod. The output mirror has a reflectivity of 90% and the rear mirror a reflectivity of 99.9%. If there is no excited state absorption (i.e., $\alpha = 0$) in the laser, what would be the fractional loss per pass in the laser cavity, and what would be the decay time of the cavity?

from (8.38)

$$L_F = \alpha L - \ln(R_1 R_2)^{1/2}$$

but $\alpha = 0$

$$\therefore L_F = -\ln(R_1 R_2)^{1/2} = -\frac{1}{2}\ln(R_1 R_2)$$

$$= -\frac{1}{2}\ln\left[(0.999)(0.90)\right] = +0.053$$

from (8.39)

$$\tau_c = \frac{nd}{cL_F} = \frac{1.76\ (0.1)}{3\times10^8\ 0.053} = 1.1\times10^{-8}\,s$$

$$= 11\ ns$$

5. What would be the maximum Q-switched power output from the ruby laser given in Problem 4 if the mode occupies a diameter of 2 mm within the rod? Assume that the laser gain is 100 times the threshold value. *Hint:* Determine ΔN_{ul} from the equations of Section 13.10.

$$\sigma_{ul} \Delta N L = \frac{1}{2} \ln\left(\frac{1}{R_1 R_2}\right)$$

$$\Delta N = \frac{1}{\sigma_{ul} L} \frac{1}{2} \ln\left(\frac{1}{R_1 R_2}\right)$$

$$M_0 = 100 \Delta N V = 100 \Delta N \frac{\pi d^2}{4} L$$

$$= 100 \pi \frac{d^2}{8} \frac{1}{\sigma_{ul}} \ln\left(\frac{1}{R_1 R_2}\right)$$

$$= 100 \frac{\pi (2 \times 10^{-3})^2}{8} \frac{1}{2.5 \times 10^{-24}} \ln\left[\frac{1}{(0.999)(0.9)}\right]$$

$$= \underline{6.68 \times 10^{18}}$$

$$P_{max} = \frac{M_0 h\nu}{2 \tau_c} = \frac{6.68 \times 10^{18} \, 6.63 \times 10^{-34} \left(\frac{3 \times 10^8}{694.3 \times 10^{-9}}\right)}{2 \, (1.1 \times 10^{-8})}$$

$$= \underline{8.7 \times 10^7 \text{ W}}$$

6. Obtain (13.23), the expression for photon density, from (13.22). Obtain the value of M where ϕ is a maximum.

$$\frac{d\phi}{dM} = \left(\frac{M_{Th}}{2M} - \frac{1}{2}\right) = \frac{1}{2}\left(\frac{M_{Th}}{M} - 1\right)$$

$$d\phi = \left(\frac{M_{Th}}{2}\frac{1}{M} - \frac{1}{2}\right) dM$$

Integrate $\Rightarrow$ $\phi = \frac{M_{Th}}{2} \ln M - \frac{M}{2} + c$

$$= \frac{1}{2}\left(M_{Th} \ln M - M\right) + c$$

when $\phi = \phi_0$, $M = M_0$

$$\therefore \quad \phi_0 = \frac{1}{2}\left(M_{Th} \ln M_0 - M_0\right) + c$$

or $c = \phi_0 - \frac{1}{2}\left(M_{Th} \ln M_0 - M_0\right)$

$$\therefore \quad \underline{\phi - \phi_0 = \frac{1}{2}\left[M_{Th} \ln \frac{M}{M_0} - (M - M_0)\right]}$$

ϕ is maximum when;

$$\frac{d\phi}{dM} = \left(\frac{M_{Th}}{2}\frac{1}{M} - \frac{1}{2}\right) = 0$$

$\frac{M_{Th}}{M} - 1 = 0$ or $\underline{M = M_{Th}}$

7. What would be the minimum pulse duration of a mode-locked alexandrite laser? If the separation between mirrors is 2 m and the alexandrite laser rod is 0.1 m long, what would be the separation between mode-locked pulses?

$d = 2\,m \qquad L = 0.1\,m \qquad n_L = 1.74$ (alexandrite)

$\Delta\nu_{FWHM} = 2.6 \times 10^{13}\,Hz \qquad n_c = 1.0$

$$\Delta t_p = \frac{1}{\Delta\nu_{FWHM}} = \frac{1}{2.6 \times 10^{13}/s} = \underline{3.85 \times 10^{14}\,s}$$

$$\Delta t_{sep} = \frac{1}{\Delta\nu} = \frac{2\left[n_c(d-L) + n_L L\right]}{c} = \frac{2\left[1(d-L) + n_L L\right]}{c}$$

$$= \frac{2\left[(2m - 0.1m) + 1.74(0.1m)\right]}{3 \times 10^8}$$

$$= 1.38 \times 10^{-8}\,s = \underline{13.8\,ns}$$

8. For a gallium arsenide distributed feedback laser in which only the index of refraction is modulated, what would be the necessary index modulation in order for the laser to operate with this type of resonator? Assume that the laser cavity is 0.5 mm in length. What would be the bandwidth of the laser output?

for index modulation, The Threshold is

$$\eta_1 = \frac{\lambda_0}{L} \frac{\ln G}{\pi G^{1/2}}$$

$L = 0.0005\,m$
$\lambda = 870\,nm$
$\eta = 3.4$

$$G = e^{2gL}$$

Let $G = 1{,}000$ from Table 15-18 pg 596

$$\text{Then } \eta_1 = \frac{\lambda_0}{0.0005} \frac{\ln 1{,}000}{\pi (1{,}000)^{1/2}}$$

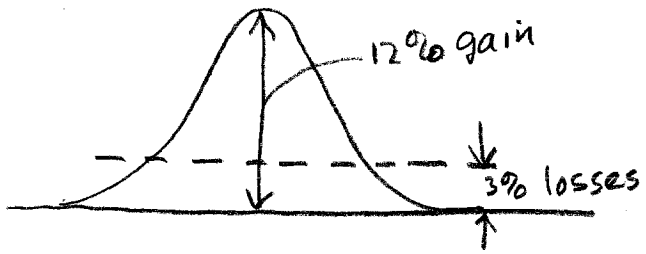
$$= \frac{870 \times 10^{-9} \ln 10^3}{5 \times 10^{-4}\ \pi\ (1000)^{1/2}}$$

$$= 3.56 \times 10^{-5}$$

$$\therefore \Delta\lambda = (3.56 \times 10^{-5})(870 \times 10^{-9}\,m)$$

$$= 3.10 \times 10^{-11}\,m = 0.031\,nm$$

9. A hypothetical gas laser operates at 200 nm in a cavity with mirrors separated by 2 m and is dominated by Doppler broadening with a Doppler width (FWHM) of 4×10^9 Hz. The gain for a single pass through the amplifier is 12%, and each mirror has a reflectivity of 98%, a transmission of 1%, and a scattering loss of 1%. There is also a scattering loss of 0.5% at each Brewster window. If this laser were mode-locked, what would be the maximum number of modes that could contribute to producing mode-locked pulses, and what would be the minimum pulse width of each pulse? What would be the separation between pulses?



$\Delta\nu_D = 4 \times 10^9$ Hz

$R = 0.98$

$T = 0.01$

$a = 0.01 + 2(0.005) = 0.02$

Total cavity losses 3%/pass

$$g^D(\nu) = \sqrt{\frac{\ln 2}{16\pi^3}} \frac{c^2 A_{ul}}{\eta^2 \nu_0^2 \Delta\nu_D} \left[N_u - \frac{g_u}{g_l} N_l\right] e^{-\left[\frac{4\ln 2 (\nu-\nu_0)^2}{\Delta\nu_D^2}\right]}$$

at $\nu = \nu_0$ $\quad g^D(\nu) = \sqrt{\frac{\ln 2}{16\pi^3}} \frac{c^2}{\nu_0^2} \frac{A_{ul}}{\Delta\nu_D}$

or $e^{-\left[\frac{4\ln 2 (\nu-\nu_0)^2}{\Delta\nu_D^2}\right]} = 1$ when gain is 12%

let $\nu - \nu_0 = \Delta\nu$ and solve when $\exp = \frac{1}{4}$ or 3%

$$e^{-\frac{4\ln 2\, \Delta\nu^2}{\Delta\nu_D^2}} = \frac{1}{4} = 0.25$$

or $e^{\frac{4\ln 2\, \Delta\nu^2}{\Delta\nu_D^2}} = 4$

$$4\ln 2 \left(\frac{\Delta\nu}{\Delta\nu_D}\right)^2 = \ln 4 = 1.386 \Rightarrow \left(\frac{\Delta\nu}{\Delta\nu_D}\right)^2 = \frac{1.386}{4\ln 2} = 0.50$$

$$\therefore \Delta\nu = \sqrt{0.50}\, \Delta\nu_D = 0.707\, \Delta\nu_D$$

$$= 0.707\,(4 \times 10^9 \text{ Hz}) = 2.82 \times 10^9 \text{ Hz}$$

$$\Delta\nu_{sep} = \frac{c}{2d} = \frac{3 \times 10^8}{2\,(2\text{ m})} = 7.5 \times 10^7 \text{ Hz}$$

$$\#\text{ modes} = \frac{2.82 \times 10^9}{7.5 \times 10^7} + 1 = 37.6 + 1 = \underline{38 \text{ modes}}$$

$$\Delta t_{sep} = \frac{2d}{c} = \underline{13.33 \text{ ns}} \qquad \Delta t_p = \frac{1}{7.5 \times 10^7\, 38} = \underline{3.5 \times 10^{-10} \text{ s}}$$

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10. You have the opportunity to observe a three-mirror laser cavity (such as that shown in Figure 13-31) in which the laser is producing mode-locked pulses at intervals of 12.0 ns. The focal length of both M_1 and M_2 is 0.1 m. What is the distance between M_2 and M_3? If the mode-locked pulses have a width of 133 fs and you know that the laser is mode-locked over the FWHM of the laser gain medium, what would you suspect is the gain medium?

for M_1 + M_2, $f = 0.1\,m$

$$\Delta t_{sep} = 12\,ns = \frac{2d}{c} \Rightarrow d = 1.80\,m$$

distance of M_2 to gain cell is $0.1\,m$

gain cell to M_1 is $2f = R = 0.2\,m$

$\therefore$ distance from M_2 to M_3 is $1.80 - 0.1 - 0.2 = 1.50\,m$

$$\Delta\nu_{sep} = \frac{c}{2d} = \frac{3\times10^8}{2(1.8)} = 8.33\times10^7\,Hz$$

$$\Delta t_p = \frac{1}{\Delta\nu_{sep} N} = 133\times10^{-15}\,sec = \frac{1}{\text{gain bandwidth}}$$

$\therefore$ gain bandwidth $= \underline{7.52\times10^{12}\,Hz}$

probably Nd:Glass with bandwidth of $7.5\times10^{12}\,Hz$